RAN-2303081201010002

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M.Sc. (Sem.-I) Examination November - 2025

Statistics : CC-Probability Theory (Paper : 101)

Time: 3 Hours] **[Total Marks: 70**

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book	
(2) Attempt all questions.	
(3) Each question carries 14 marks.	
(4) Answer any two out of the three bits given in each question.	Student's Signature

Q.1. (a) Define a random variable. Prove any of its two properties.

(b) Define independence of random variables and show that E(XY) =E(X)E(Y) if the random variables are independent. Show that the converse of this statement is false.

(c) State and prove Holder's inequality and Minikowaski inequality.

Q.2. (a) Write a detailed note on stochastic independence.

(b) Define a Tail σ-field and state and prove Kolmogorov's zero -one law.

(c) State and prove the inversion Theorem.

Q.3. (a) Explain : (i) Convergence in r^{th} mean
(ii) Convergence in Distribution
(iii) Almost everywhere convergence
State and prove their inter relationships.

(b) Discuss conditional expectation of a random variable and prove any of its two properties.

(c) What do you understand by weak Law of Large Numbers? State and prove any two WLLN.

Q.4. (a) Find the characteristic function for Cauchy distribution. Discuss its importance

(b) Define a discrete distribution function. Show that a distribution function can have only a countable number of points of discontinuity.

(c) Decompose the following function into discrete and continuous.

i. $$F(x)=\begin{cases} 0 & if \quad x<-1 \\ \dfrac{x+1}{5} & if \ -1\le x<1 \\ 1-\left(\dfrac{1}{2}\right)^{x} & if \quad x\ge 1 \end{cases}$$

ii. $$F(x) = \begin{cases} 0 & \text{if} \quad -2 \le x < 0 \\ 1/4 & \text{if} \quad 0 \le x < 2 \\ 1 - \frac{1}{2} e^{-x} & \text{if} \quad x \ge 2 \end{cases}$$

Q.5. (a) i. If $\{A_n\}$ is a sequence of events such that $A_1 = \Omega$, and $P(A_n) = \frac{1}{2} P(A_{n-1})$ for $n = 2...$ Find $P(\mathit{Lim\ Sup}\ A_n)$.

ii. Let $\{X_n\}$ be a sequence of independent random variables with $P(X_n = \pm n) = 0.5$. Show that SLLN does not hold for the sequence.

(b) Let $\{x_n\}$ be a sequence of independent random variables with $E(X_n) = \mu_n$, $E(X_n - \mu_n)^2 = \sigma_n^2 \neq 0$ and $E|X_n - \mu_n|^{2+\delta}$ exists for each n where $\delta > 0$. Let $S_n = \sum_{i=1}^{n} X_i$ and $V(S_n) = s_n^2 = \sum_{i=1}^{n} \sigma_i^2$ then if $\frac{1}{S_n^{2+\delta}} \sum_{i=1}^{n} E|X_i - \mu_i|^{2+\delta} \to 0$ as $n \to \infty$ show that $\frac{S_n - E(S_n)}{S_n} \xrightarrow{D} Z \sim N(0,1)$

(c) Discuss conditional dominated convergence theorem.
